**Bifurcations / Hysteresis**

**Saddle-node bifurcation:** (fold bifurcation)

\[ \frac{dx}{dt} = r + x^2 \]

- \( r < 0 \):
  - Stable
  - Unstable

- \( r = 0 \):
  -\( x = \pm x \)

- \( r > 0 \):
  - No solutions

\( \frac{dx}{dt} = 0 \)

\( r = -x^2 \)

\( r > 0 \Rightarrow \) no solutions \( \frac{dx}{dt} = -x \)

\( r < 0 \Rightarrow \) \( x = \pm x \)

**Fixed points are created and destroyed as \( r \) is varied.**

\( r \sim \text{Degradation rate} \)

\( \text{Binding affinity} \)

\( \text{Salt concentration} \)

**Transcritical bifurcation:**

\[ \frac{dx}{dt} = r x - x^2 \]

- \( r < 0 \):
  - Stable
  - Unstable

- \( r = 0 \):
  -\( x = \pm x \)

- \( r > 0 \):
  - No solutions

\( \frac{dx}{dt} = 0 \)

\( r = x \)

\( r < 0 \Rightarrow \) \( x = \pm x \)

**Exchange of stabilities**

\[ \frac{dx}{dt} = -r x - x^2 \]
Pitchfork Bifurcation (supercentral)
\[ \frac{dx}{dt} = rx - x^3 \] (invariant under \( x \to -x \))

Fixed Points appear/disappear in symmetrical pairs

Critical slowing down

\[ 0 = rx - x^3 \]
\[ r = x^2 \]
\[ x = \sqrt{r} \]

Exploding instability

Computer example:
\[ \frac{dx}{dt} = rx + x^3 \] (itself not)

\( r \leq 0 \) unstable
\( r = 0 \) stable
\( r > 0 \) unstable

Hysteresis