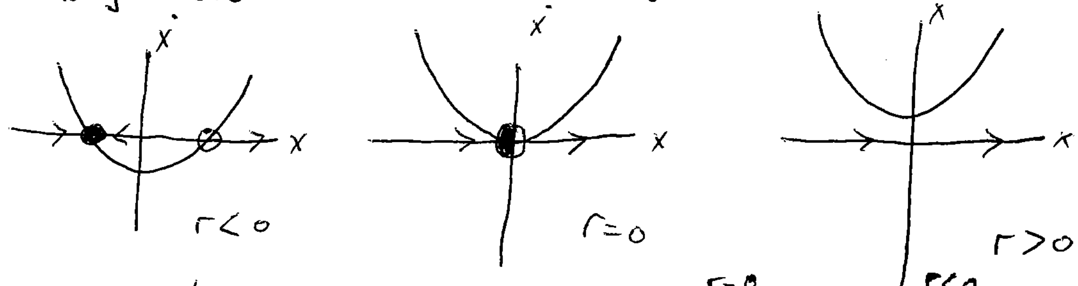
Bifurcations / Hysteresis

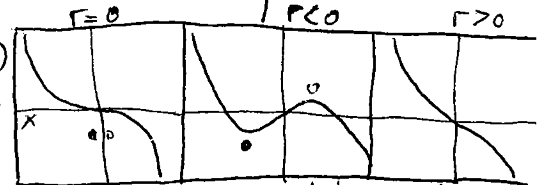
saddle-node bifurcation : (fold bifurcation).

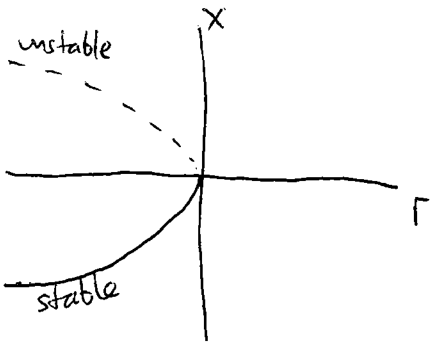
$$\frac{dx}{dt} = r + x^2$$



r = control parameter

think of r as independent parameter





$$\frac{dx}{dt} = 0$$

$$r = -x^2$$

$r > 0 \longrightarrow$ no solutions

$r < 0 \longrightarrow x = \pm \alpha$

$$f(x) = -\frac{dV}{dx}$$

map of how fixed points behave as you change r

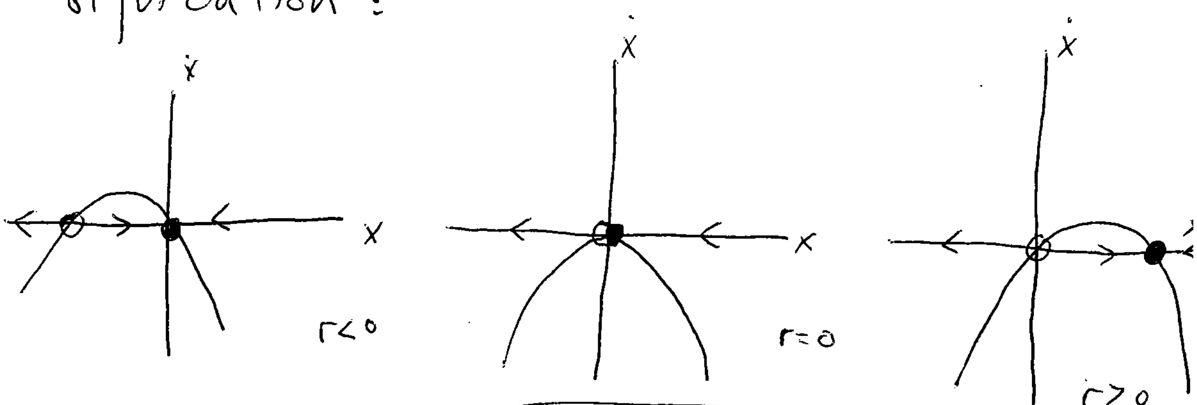
BIFURCATION DIAGRAM

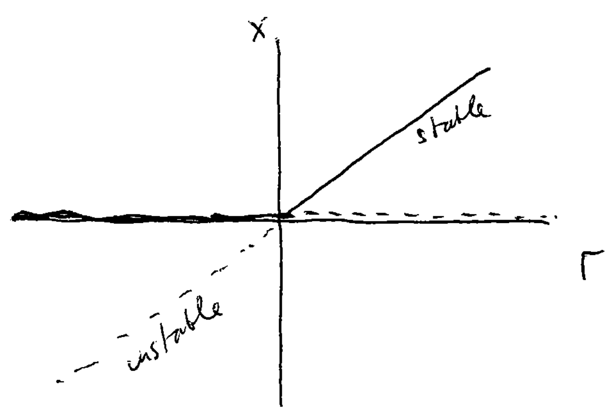
FIXED POINTS ARE CREATED AND DESTROYED AS r IS VARIED.

r ~ DEGRADATION RATE
BINDING AFFINITY
SALT CONCENTRATION

transcritical bifurcation :

$$\frac{dx}{dt} = rx - x^2$$



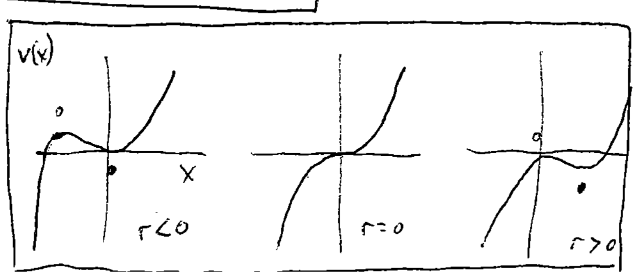


exchange of stabilities

$$\frac{dx}{dt} = 0$$

$$-rx = -x^2$$

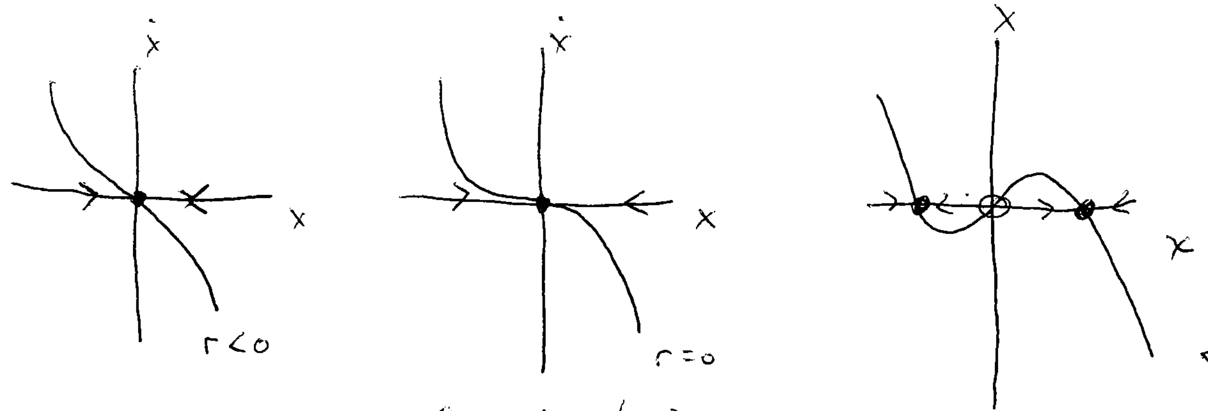
$$r = x$$



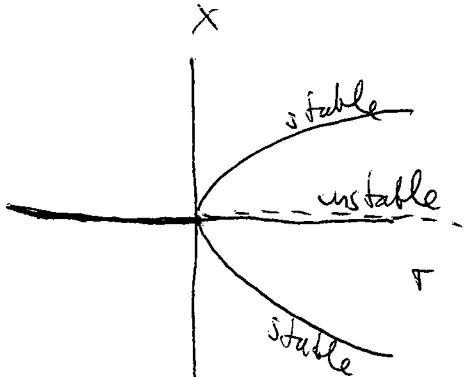
Pitchfork Bifurcation: (supercritical)

$$\frac{dx}{dt} = rx - x^3$$ (invariant under $x \to -x$)

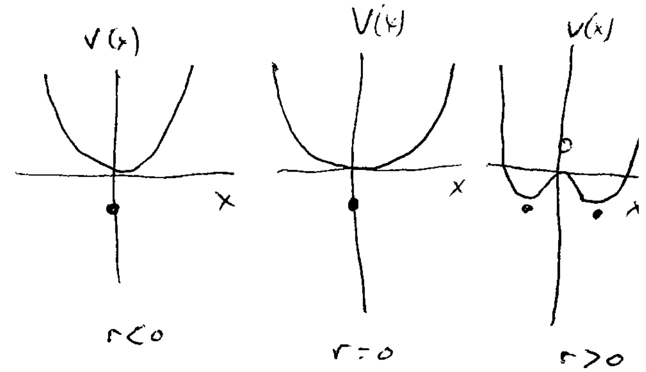
(symmetry) FIXED POINTS APPEAR/DISAPPEAR IN SYMMETRICAL PAIRS



(critical slowing down)

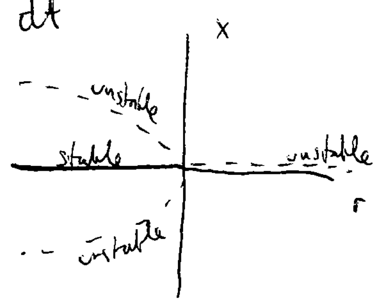


$$0 = rx - x^3$$
$$r = x^2$$
$$x = \sqrt{r}$$

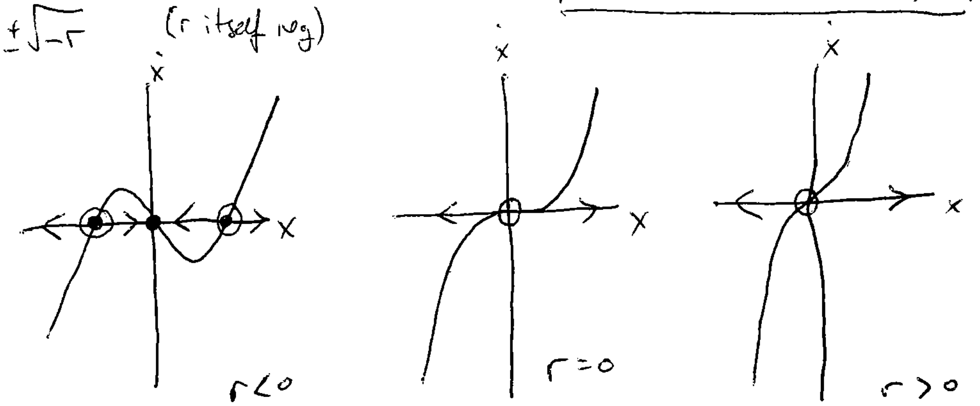


$$\frac{dx}{dt} = rx + x^3$$ subcritical pitchfork bifurcation

EXPLOSIVE INSTABILITY



$x^* = \pm\sqrt{-r}$ (r itself neg)



Computer example:

$$\frac{dx}{dt} = rx + x^3 - x^5$$ → removes explosive instability